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# ON THE MOTION OF A PLANET, ASSUMING THAT THE VELOCITY OF GRAVITY IS FINITE.

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Laplace, in the *Mécanique Céleste* (Tome X, Art. 22), considers the hypothesis, that gravitation is produced by the impulsion of a fluid toward the centre of the attracting body, and finds that to account for the acceleration of the moon's mean motion the velocity of the gravific fluid must be at least one hundred million times that of light. He adds :

“ Il est aisé de voir que l'équation seculaire de la terre, due à la transmission successive de la gravité, n'est qu'un sixieme environ de l'équation correspondante de la lune, et par conséquent, elle est nulle ou insensible.”

Although the value of the acceleration employed by Laplace was erroneous, the argument is not affected thereby.

Lehmann-Filhés (*Astronomische Nachrichten*, No. 2630) considers the hypothesis that gravity has a finite velocity of propagation ; but his hypothesis differs from that of Laplace in that no fluid is assumed. I propose in this note to consider the same hypothesis, retaining the terms in  $\lambda m$  which he neglects, but assuming that no secular disturbances are produced by the motion of the solar system through space. Let  $\xi_0, \eta_0$  and  $\xi, \eta$  be the rectangular co-ordinates of the sun and the planet, in the plane of the orbit, referred to fixed axes. Also put

$$r = \sqrt{(\xi - \xi_0)^2 + (\eta - \eta_0)^2} = \sqrt{x^2 + y^2},$$

in which  $r$  is the distance between the two bodies, and

$$x = \xi - \xi_0, \quad y = \eta - \eta_0$$

are the co-ordinates of the planet with reference to the sun.

Let  $m$  be the mass of the planet, taking the sun's mass as the unit.

If we assume that gravity requires an interval  $\lambda(1+m)r$  to pass from the sun to the planet and *vice versa*, the co-ordinates of the sun at the time the impulse starts therefrom are, to terms of the first order,

$$\xi_0 - \lambda(1+m)r \frac{d\xi_0}{dt}, \quad \eta_0 - \lambda(1+m)r \frac{d\eta_0}{dt}, \quad (1)$$

in which the differential co-efficients are those obtained upon the assumption that gravity acts instantaneously; i. e.

$$\begin{aligned} m \frac{d\xi}{dt} + \frac{d\xi_0}{dt} &= 0, & \frac{dx}{dt} &= (1+m) \frac{d\xi}{dt} = - \left( 1 + \frac{1}{m} \right) \frac{d\xi_0}{dt}, \\ m \frac{d\eta}{dt} + \frac{d\eta_0}{dt} &= 0, & \frac{dy}{dt} &= (1+m) \frac{d\eta}{dt} = - \left( 1 + \frac{1}{m} \right) \frac{d\eta_0}{dt}; \end{aligned}$$

whence the quantities (1) become

$$\xi_0 + \lambda m r \frac{dx}{dt}, \quad \eta_0 + \lambda m r \frac{dy}{dt}.$$

The distance of the planet from the point thus determined is

$$r \left( 1 - \lambda m \frac{dr}{dt} \right),$$

and the corresponding rectangular components,

$$x - \lambda m r \frac{dx}{dt}, \quad y - \lambda m r \frac{dy}{dt}.$$

The resulting equations of motion for the planet are

$$\begin{aligned} \frac{d^2\xi}{dt^2} &= -k^2 \frac{x - \lambda m r \frac{dx}{dt}}{r^3 \left( 1 - \lambda m \frac{dr}{dt} \right)^3} = k^2 \left[ -\frac{x}{r^3} + \frac{\lambda m}{r^2} \frac{dx}{dt} - 3 \frac{\lambda m x}{r^3} \frac{dr}{dt} \right], \\ \frac{d^2\eta}{dt^2} &= -k^2 \frac{y - \lambda m r \frac{dy}{dt}}{r^3 \left( 1 - \lambda m \frac{dr}{dt} \right)^3} = k^2 \left[ -\frac{y}{r^3} + \frac{\lambda m}{r^2} \frac{dy}{dt} - 3 \frac{\lambda m y}{r^3} \frac{dr}{dt} \right]. \end{aligned}$$

The co-ordinates of the planet at the time the impulse starts therefrom toward the sun are

$$\begin{aligned} \xi - \lambda (1+m) r \frac{d\xi}{dt} &= \xi - \lambda r \frac{dx}{dt}, \\ \eta - \lambda (1+m) r \frac{d\eta}{dt} &= \eta - \lambda r \frac{dy}{dt}. \end{aligned}$$

The distance of the sun from the point thus determined is

$$r \left( 1 - \lambda \frac{dr}{dt} \right);$$

the corresponding rectangular components,

$$-x + \lambda r \frac{dx}{dt}, \quad -y + \lambda r \frac{dy}{dt};$$

and the equations of motion,

$$\begin{aligned} \frac{d^2 \xi_0}{dt^2} &= k^2 m \frac{x - \lambda r \frac{dx}{dt}}{r^3 \left[ 1 - \lambda \frac{dr}{dt} \right]^3} = k^2 \left[ \frac{mx}{r^3} - \frac{\lambda m}{r^2} \frac{dx}{dt} + 3 \frac{\lambda m x}{r^3} \frac{dr}{dt} \right], \\ \frac{d\eta_0}{dt^2} &= k^2 m \frac{y - \lambda r \frac{dy}{dt}}{r^3 \left[ 1 - \lambda \frac{dr}{dt} \right]^3} = k^2 \left[ \frac{my}{r^3} - \frac{\lambda m}{r^2} \frac{dy}{dt} + 3 \frac{\lambda m y}{r^3} \frac{dr}{dt} \right]. \end{aligned}$$

Subtracting (3) from (2), we have

$$\begin{aligned} \frac{d^2 x}{dt^2} + \frac{k^2 (1+m)x}{r^3} &= \frac{2k^2 \lambda m}{r^2} \frac{dx}{dt} - \frac{6k^2 \lambda m x}{r^3} \frac{dr}{dt} = X, \\ \frac{d^2 y}{dt^2} + \frac{k^2 (1+m)y}{r^3} &= \frac{2k^2 \lambda m}{r^2} \frac{dy}{dt} - \frac{6k^2 \lambda m y}{r^3} \frac{dr}{dt} = Y, \end{aligned}$$

in which  $X$  and  $Y$  are the disturbing forces due the velocity of gravity.

The resulting disturbing forces in the direction of the radius vector and perpendicular thereto are

$$\begin{aligned} R &= \frac{x}{r} X + \frac{y}{r} Y = -\frac{4k^2 \lambda m}{r^3} \left( x \frac{dx}{dt} + y \frac{dy}{dt} \right) = -\frac{4k^3 \lambda m}{r^2 \sqrt{p}} e \sin v, \\ S &= \frac{x}{r} Y - \frac{y}{r} X = \frac{2k^2 \lambda m}{r^3} \left( x \frac{dy}{dt} - y \frac{dx}{dt} \right) = \frac{2k^3 \lambda m \sqrt{p}}{r^3}, \end{aligned}$$

in which  $p$  is the semi-parameter,  $e = \sin \varphi$  the eccentricity, and  $v$  the true anomaly.

The motion of the perihelion is given by the equation

$$\begin{aligned} \frac{d\pi}{dt} &= \frac{1}{ek\sqrt{p}} [-p \cos v \cdot R + (p+r) \sin v \cdot S] \\ &= \frac{2k^2 \lambda m}{r^2} \left( \sin 2v + \frac{p+r}{er} \sin v \right), \end{aligned}$$

the constant part of which is

$$\left[ \frac{d\pi}{dt} \right] = 0.$$

The variation of the mean motion in longitude is given by the equation

$$\begin{aligned}\frac{dn}{dt} &= -\frac{3an}{k\sqrt{p}} \left( e \sin v \cdot R + \frac{p}{r} \cdot S \right) \\ &= \frac{6k^2\lambda man}{pr^2} \left( 2e^2\sin^2v - \frac{p}{r^2} \right); \end{aligned}$$

whence, for the constant part, neglecting terms in  $e^2$ ,

$$\left[ \frac{dn}{dt} \right] = -6n^3\lambda m.$$

The corresponding term in the mean longitude is

$$\Delta L = -6n^3\lambda m (t - t_0)^2,$$

in which  $t - t_0$  is the interval of time over which the integration is performed.

If we assume as the unit of time,  $\tau$ , the interval required for one revolution of the planet about the sun, we have  $n\tau = 2\pi$ ; whence

$$\Delta L = -48\pi^3\lambda m (t - t_0)^2.$$

In the case of the earth, if we assume  $t - t_0 = 2000$  years, and that the velocity of gravity is the same as that of light, we obtain

$$\Delta L = -48\pi^3 \times \frac{500}{365\frac{1}{4} \times 86,400} \times \frac{4,000,000}{330,000} \times 57^\circ.3 = -16^\circ.4,$$

a quantity far in excess of errors of observation.

Nevertheless it cannot be assumed as proved that gravity is not due to *longitudinal* waves in the luminiferous ether, since the velocity of these may be much greater than the transverse waves which we recognize as light and electricity.

The hypothesis here considered does not explain the secular inequality in the moon's motion, since the value of  $\Delta L$  found above is negative, whereas in the case of the moon it should be positive.